ARE WE THERE YET? AN OBJECTIVE MECHANISM TO SUPPORT THE ASSESSMENT OF DRIVING SIMULATOR UTILITY

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Abstract – Virtual prototyping using driving simulators offers a highly cost effective alternative to test track evaluations. A pressing question asked by car manufacturers is what level of simulator fidelity is needed for evaluating a particular vehicle, vehicle sub-system, driver control interface, or driver infotainment system. This paper adopts a driver modelling perspective to addresses this question and defines a process based on a simulator utility-triplet to establish whether a simulator yields absolute behavioural fidelity for a particular driving task. The adopted driver model is a cybernetic cascade model that includes perception of multi-modal cues that drivers use to assess vehicle state relative to the environmental constraints. These multi-modal cues in the simulator are perceived through the particular simulator's rendering transfer function that may cause driver adaptations to yield the desired performance level. By exploring the degree to which model coefficients differ from those observed in reality across a number of basis-tasks, an objective assessment is established to objectively compare and contrast different prototype evaluation environments.

Key words: Cybernetic Driver Model, Simulator behavioural Fidelity, Driver Performance Assessment, Cue Rendering, Virtual Prototyping.

1. Introduction

Research driving simulators are commonly used to facilitate scientific evaluations of driver behaviour. Whilst designers of such simulators strive to reproduce high quality visual, vestibular, proprioceptive and auditory cues within their facilities, both financial and technological constraints limit a simulator's capability to fully recreate a real driving environment. When established [e.g. Bel1], demonstration of a driving simulator's relative behavioural validity [Bla1] justifies its use in driver behavioural studies [Kap1].

A much less frequent use of driving simulation is in the support of vehicle design. Such virtual prototyping has the advantage of reducing the need to develop expensive physical vehicle prototypes, but carries with it the unenviable burden of guaranteeing the simulator's absolute behavioural fidelity. However, with a task-based approach, this burden becomes realistic. The nature of vehicle design is that a number of specific driving manoeuvres define typical objective evaluations of a vehicle's performance, ride, handling and stability characteristics. In effect, this allows a corresponding task-based assessment of simulator fidelity, not unlike the competency-based approach of the International Civil Aviation Organisation's (ICAO, 2010) published recommendation (ICAO 9625) to National Aviation Authorities to regulate member state's use of Flight Simulation Training Devices (FSTD).

In order to tie the technical requirements of a FSTD more closely to the level of pilot training or skills assessment required, ICAO 9625 provides a mapping between a FSTD's characteristics and the associated training that may be performed with devices having such features. This certification is made against a list of tasks dictated by the procedural and methodical nature of commercial pilot training and skills evaluation. By defining the tasks required of the pilot, the demands required of the simulator itself are more easily identified. With this task classification central, it becomes possible to define acceptable simulator

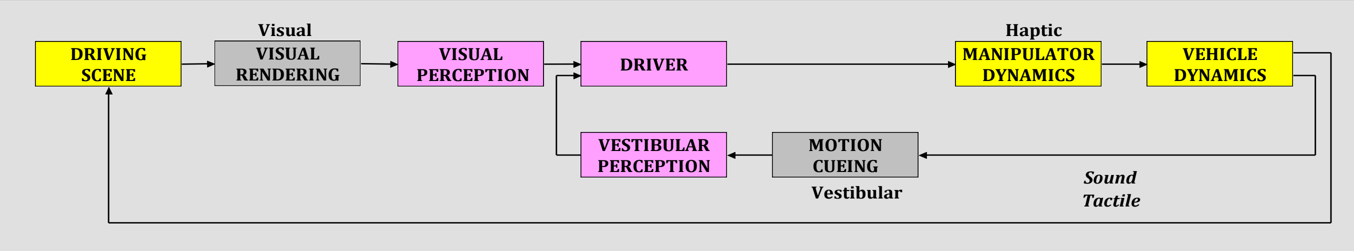

Figure 1. Driver's perception (magenta boxes) in a simulator takes place through cue rendering transfer functions (grey boxes). It is assumed that yellow boxes can be represented veridical in a simulator because those are not the most cost consuming simulator components; motion and visual system are most costly.

characteristics by assessing the ability of the FSTD to support flight crew training/assessment within the operational range of the simulator defined by those characteristics.

In contrast, assessing the merits of a particular car design is generally the responsibility of the vehicle manufacturer, whose interest is in understanding the implications of minor changes in vehicle stability, suspension, assist systems, body re-design, etc. Typical assessments require test drivers to evaluate safe, agreeable and controlled operation of the vehicle based on a perception of the entire driving environment. Theoretically, defining this plethora of tasks in order to, in turn, define an acceptable driving simulator operational range is possible. The cardinal challenge, however, lies in defining "acceptable" fidelity; the focus of this paper.

1.1. Simulator Utility Quantification

Cybernetic driver models have been used to quantify how drivers perceive cues and integrate them to produce vehicle control actions. Even through most tend to exist only for lateral, lane-keeping manoeuvres [see Ste1 for a recent review], such models have previously been used to objectively assess the design of driving simulators for curve negotiation tasks [Dam1].

The drawback of existing driver models is that few explicitly model driver's perception and integration of available cues. Such a low-level perception model is needed to understand how the particular cue-rendering employed by the simulator influences driver behaviour. Figure 1 shows that simulators add extra dynamics into the perception-action loop in the form of cue-rendering transfer function (grey boxes) and therefore force the driver to adapt in order to maintain equivalent performance. To be able to understand and predict the effect of different simulator cue rendering techniques, we need to know how drivers use these cues at the lowest level. We recommend that simulator developers characterize their simulators based on cue rendering transfer functions as a means to report objective simulator cue-fidelity.

Under normal driving conditions drivers learn to use the available multi-modal cues when they provide a benefit to control performance and stability [Pas1]. In the cascade control of the cybernetic driver model, feedback loops of vehicle position, velocity, acceleration and steering torque are present. The driver perceives these different cue channels with different sensory organs and these cues are rendered by the simulator with different transfer functions. Thus if a channel is removed such as motion or steering torque, then those vehicle states can no longer be directly perceived and have to therefore be estimated based on derivatives of other cues. For example, acceleration, which is normally perceived directly with the vestibular organ, can also be perceived by taking derivatives of visual cues. Taking derivatives requires sampling, introducing a delay. Thus removal of a cue that offers direct perception of a vehicle state will introduce a delay in one of the feedback loops. A delay in one of the feedback loops results in less stability can be observed in these data as larger control actions, more corrective control actions and a more intermittent control. The removal of cues shows its effect especially in high bandwidth manoeuvres such as a chicane or double lane change for which visual cues alone are not fast enough to yield sufficient stability [Hos1].

While existing models [Ste1] are certainly suitable to show differences in behaviour caused by different cue rendering techniques by showing different model coefficients for different cue rendering techniques [Dam1], they cannot predict the effect of such different rendering techniques a-priori.

The ultimate goal of our cybernetic driver model is that it can be used as a standalone virtual prototyping tool and for that it is necessary to explicitly model cue perception and integration as well as the performance goals and adaptation mechanisms that drivers use to adapt their behaviour so that the model produces emergent behaviour that matches what is observed in reality. This long term self-organizing model goal will not be discussed any further in this paper.

Driving is not simply a combination of open and closed loop controllers; it also requires learning of more complex control profiles to be able to perform complex manoeuvres such as parking or optimal obstacle avoidance. These manoeuvres cannot be modelled as simple stimulus response controllers but require at least integration of information up to some horizon and shape the control profile to optimize some performance criterion. Normally, models based on optimal preview control [Tom1] are used or a more modern approach based on model predictive control [Kee1]. Our current scientific challenge is to combine perceptual driver models with cue integration and model predictive control models with explicit performance optimization such that the resulting cybernetic model can predict effects of changes in cue fidelity (effect of simulator) or the introduction of a new support system (effect of vehicle).

The current paper focusses on the first practical steps that will ultimately aid car manufacturers to know whether a simulator is suitable for evaluating a particular prototype. For this, we present a mechanism to quantify a simulator's behavioural fidelity.

2. Method

By focusing on just one of the many manoeuvres defined in a typical vehicle's verification programme, vehicle handling through a double lane-change or chicane [ISO 3888-2], this paper attempts to combine ICAO's competency-based approach with cybernetic driver modelling to define a methodology to assess simulator utility for this specific task in virtual vehicle prototyping.

2.1. Cybernetic Driver Model

A cybernetic driver model has been established to describe the transfer functions that map cues (perceptions in the simulator) to control (handling of the vehicle). Focus is directed primarily to visual, vestibular and haptic cues but other domains are also recognized as potentially important in yielding realistic driver behaviour in simulators. The model under development is a cascade controller based on the classic cross-over model applied to vehicle handling (STI Driver Model – McR1), but expanded to both lateral and longitudinal vehicle handling. A simplified version is depicted in Figure 2 for the purpose of highlighting the various components that make up the current version of the model. The model in its current incarnation is a pure open plus closed loop model and does not explicitly include usage of the entire preview nor optimize an extended control profile as is done in optimal control or model predictive control models. These elements are being added later.

The proposed cybernetic model not only models the dynamics of the visual, vestibular and neuro-muscular sensory organs but also their integration into a single internal representation of relevant vehicle states. Most existing models simply assume direct

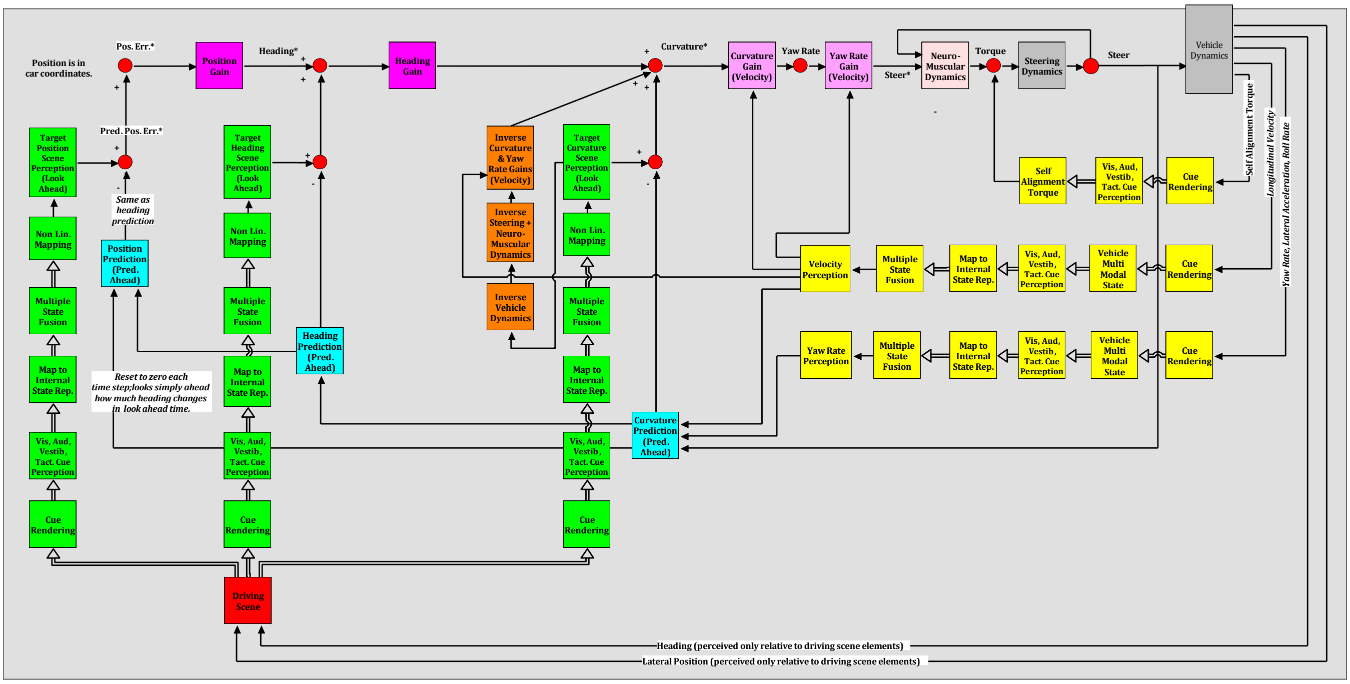

Figure 2. Cybernetic driver model for lateral-control only (for space reasons). See text for detailed explanation of the adopted colour scheme.

perception of these vehicle states and show how model gains change under different conditions but such models do not provide an explicit explanation of the mechanism that results in the final model coefficients; as discussed earlier, such models cannot be expected to predict effects of changes in cue rendering or be used to optimize cue-rendering strategies.

The model in Figure 2 shows several types of boxes each indicated by a different colour:

- The perception of absolute vehicle state fundamentally influenced by cue rendering and cue perception (yellow).
- The perception of relative vehicle state, based primarily on visual environmental cues including preview (green).
- Feed-forward open loop control, representing a driver's internal model relating vehicle control actions to vehicle dynamics (orange).
- Prediction, based on look-ahead time to equalise lags and delays in human/vehicle system (cyan).
- Proprioceptive feedback, modelled as the coupling between the neuromuscular part of the driver and the pedal/steering manipulator dynamics (pink).
- Non-linear control, described as a component of each environmental state perception branch that feeds into the feed forward control path as well as into the error calculation of perceived minus predicted curvature / acceleration (green).

A fully detailed exposition of the model is beyond the scope and spatial constraints of the paper. Here it suffices to state that the sketched cybernetic model is needed for modelling driving manoeuvres such as stopping, lane changing, and obstacle avoidance. The vast majority of existing model focus on car following and gentle radius curve negotiation which can be modelled well without resorting to optimal preview models and a strong use of internal models because of the relatively low bandwidth of the conditions generally studied.

As stated earlier, the target of our cybernetic model development is for it to produce realistic driver behaviour from first principles rather than through explicit identification of model coefficients. In other words, the model should through self-organizing optimization of the balance between performance/safety and effort, be able to match observed driver behaviour in reality (no cu-rendering) and simulator (with cue rendering). An example of such a model for car following in fog is detailed in [Car1]. However, a general full blown self-organizing model is still years removed from maturation but the current research program at U-Leeds with extensive studies comparing real world and simulator driving across a set of basis driving tasks with a limited group of skilled drivers performing all real world and simulator trials is well situated to establish such a model.

2.2. Driving Manoeuvre/Task

The specific driving task targeted in the remainder of this paper, a double lane, is shown in Figure 3. The task was performed in two driving environments with a single driver, one driver in reality and one driver in the virtual conditions of the University of Leeds Driving Simulator, UoLDS [Jam1]. The total length of the high speed chicane (orange dots) is 50m which is half a sinusoid in steering. At 60kph or 17.14mps takes about 3s. Speed was controlled automatically in the UoLDS.

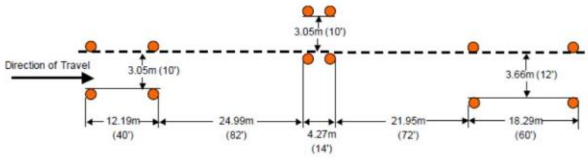

Figure 3: Double lane change cone placement as replicated in the UoLDS.

In reality, the task was performed by a Jaguar Land Rover test-driver in a XF model vehicle on a frozen lake on a proving ground in Sweden. The recorded coefficient of friction of the surface was 0.3 and the task was performed at 60kph without any active vehicle control systems enabled. The driver was instructed to produce the desired target speed which test drivers are highly skilled at.

The same conditions were recreated in the UoLDS and the single driver, experienced with the handling of the simulator under full feedback configuration, undertook the task in three conditions, repeating each four times:

- *Baseline*: simulator operational with full motion and steering torque feedback.
- *NoSteer:* motion system on but without any steering feel provided to the driver.
- *NoMotion:* steering feedback on but without any inertial cues provided to the driver by the motion system.

Here we only report on the performance in the simulator and leave model fitting to a follow-up paper. The main reason for this choice is that the behaviour in the simulator shows fundamentally different behaviour as we discuss in section 4 below.

2.3. Utility Score of Behavioural Fidelity

Behavioural fidelity in this context is defined as the degree to which behaviour in the simulator (UoLDS) during the double lane-change is statistically indistinguishable from the test-track. Evaluation of behavioural fidelity through time-series comparison alone lacks the conclusive insight into the effect of the driving simulator system on the driver. For example, the same level of performance can be achieved with different levels of effort depending on how easy it is to control the vehicle. Hence, three levels of behavioural fidelity assessment are defined, the Utility Triplet.

- *Aggregate Performance* looks at specific metrics that can be extracted from the vehicle state or driver control that quantify performance, risk or effort. This includes spatial and temporal proximity to constraints as well as completion time. Focus is placed on accuracy or the degree to which the task was performed and includes metrics such as standard deviation of steering angle or steering rate. These aggregate metrics can be computed from the available signals without any special decision logic. These aggregate metrics do not really show when and how control is applied.
- *Time Series Comparison* profile driver control actions as a function of time and distance in the manoeuvre, showing when and how control is applied in order to gain insight into the specifics of control. Examples of these metrics are peak to peak analysis in steering rate such as number and magnitude of corrective steering actions, or the lag between control actions and specific vehicle states. These metrics require development of signal specific logic to extract the meaningful time series related metrics.
- *Transfer Function* analyses place focus on perceptual input to control actions rather than vehicle movements, predominantly as time series metrics do not really show what caused the production of the specific control signal profile. In case of the current study, the cybernetic driver models combines open and closed loop control, as well as hard code the availability of different feedback channels (e.g. haptic or vestibular). Human perception and execution are necessarily noisy and thus errors build up even if the driver has a perfect internal model of the environmental constraints and the vehicle. In theory, the driver adapts to yield maximal performance at minimal effort; if this mechanism can be explicitly modelled, then we have made a significant step not only in understanding human drivers but also towards the use of models in virtual prototyping. The metrics here are model coefficients together with estimates of standard error as obtained using the statistical bootstrap method.

To judge a simulator's utility for virtual prototyping, the metrics of all three elements of the utility triplet have to be statistically equivalent; i.e. within normal behavioural variability within and between drivers.

3. Results

Behaviour in the three simulator conditions in the simulator is analysed to establish the impact of motion vestibular feedback and manipulator torque feedback on performance of a double lane change at 60kph. The subject performed each condition 4 times but we only show the last two trials to allow for some behaviour adaptation in the subject.

3.1. Aggregate Performance

The chicane task was performed according to specifications; i.e. no cones were clipped. We also saw that the lower maximum lateral acceleration was observed with full feedback than with either no motion or no steering feedback but, as is often the case, those differences were not as substantial as those observed in the manner in which the driver steered the vehicle to achieve these task performance levels.

3.2. Time Series Comparison (Profiles)

The trajectories in Figure 4 show that they are clustered per condition. We see that the driver steers back sooner when realistic feedback is missing possibly because they over-steer in the first place. Such over-steering is reasonable given the fact that the driver may have expected a build-up of vestibular or steering torque signal that he normally may use to gauge rate and timing of steering actions.

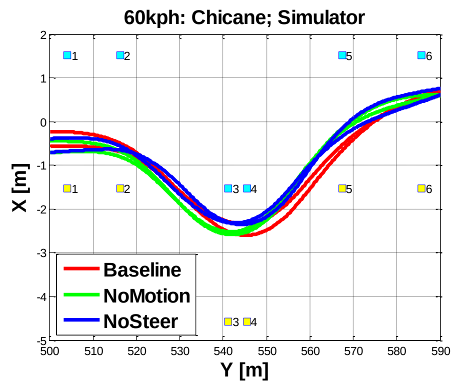

Figure 4. Trajectory of chicane in three simulator configuration conditions.

Such usage of vestibular and haptic cues is not expected to adapt away in 4 trials. Naturally, when the driver is given ample time to adapt, he/she will adopt a new driving strategy that does not rely on those cues. Here we looked at the effect of removing cues on performance.

The steering profiles in Figure 5 do indeed show that the driver steers into the manoeuvre more aggressively (higher peak and rate around 530m) and also steers back more aggressively (around 555m). Because of the overly rapid and strong steering control, the vehicle is perturbed more and the driver has to work harder to correct and re-stabilize the car which is also clearly seen in the greater peaks in the steering rate (bottom panel of Figure 5).

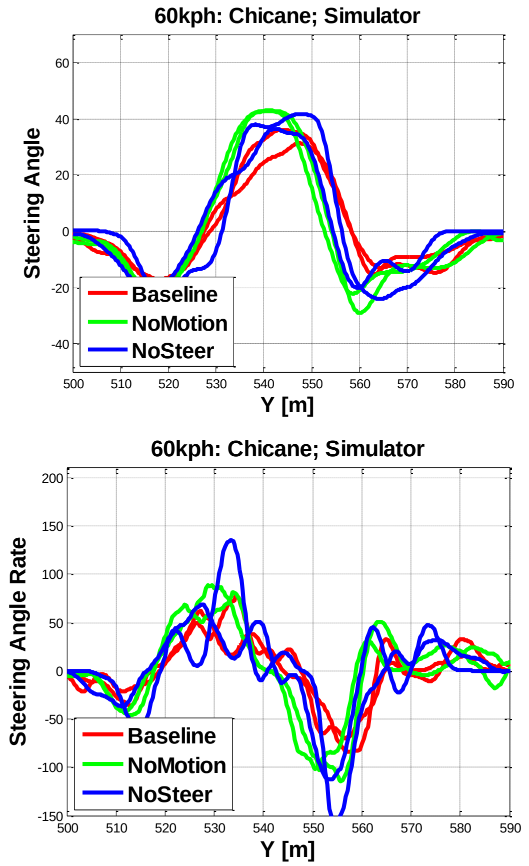

Figure 5. Steering (top) and steering-rate (bottom) profiles as a function of distance down the main axis of the lane change (y-world).

3.3. Time Series Comparison (Metrics)

In order to put quantitative metrics on these time series observations, several metrics specific for the chicane were developed. A sensitive metric is the peak to peak (peak2peak) behaviour in the steering rate because it shows the magnitude of actions (necessary to make the double lane change) and corrections (necessary to stabilize the vehicle) as well as the number of control actions and corrections made. Because we are looking at simulator data, no sensor noise is added and therefore no filtering is needed to eliminate steering rate peaks caused by noise. The resulting metrics are described next and shown in Figures 6 and 7:

- Steering Rate STD (not shown) showing the total power in steering actions and corrections that the driver applies.
- Sum of all absolute magnitudes between successive peaks in the steering rate signal (Figure 6).
- Relationship between the median steering action/correction and the number of steering action/corrections (figure 7).

Figure 6 clearly shows that the total steering-rate-power (std) increases when valuable cues are removed. No motion feedback and no steering feedback both show increased corrective control power as if open loop control was no-longer as effective; this is expected because the driver was not given time to settle into a new open loop control strategy without using motion or steering feedback.

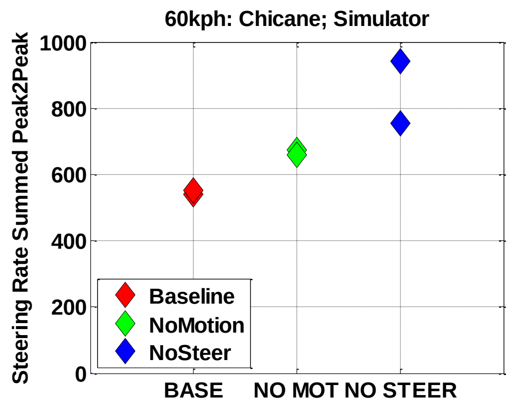

Figure 6. Summation of all absolute peak2peak magnitude changes in steering rate.

Figure 6 shows that the total sum of the absolute value of all the steering corrections (i.e. peak2peak magnitudes in steering rate signal). This does not integrate over time as for an standard deviation but over the number of steering peaks. Figure 6 shows that the NoSteer (blue) condition lies fully above the NoMotion (green) condition suggesting that the driver performs more frequent large corrections without the steering feedback than without the motion feedback; probably because the steering feedback loop is fastest.

Figure 7 shows that the number of steering corrections is highest and weakest when motion and steering feedback are available (red) and that corrections are stronger and less frequent when feedback cues are removed.

This particular fact is discussed below in the context of the cybernetic driver model.

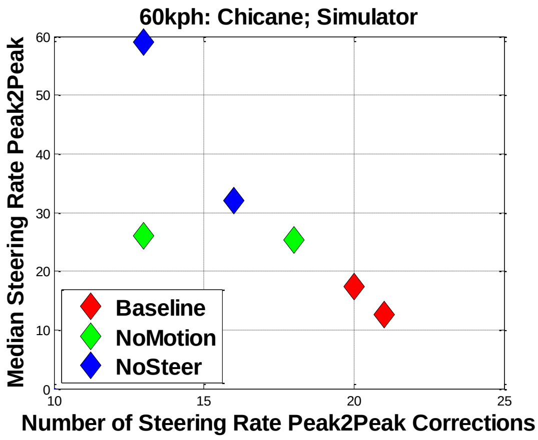

Figure 7. Median absolute peak2peak magnitude changes vs number of peak2peak steering corrections.

3.4. Transfer Function

A most compact and informative quantification of behaviour is through estimation of model coefficients or cost function weights. As detailed in the discussion, further model development to include optimal preview control for a discrete open loop manoeuvre task such as the chicane is needed which remains to be done.

4. Discussion

Metrics should be an embodiment of those task aspects that drivers take into consideration in the learning/adaptation process of arriving at an acceptable control strategy. They are the performance metrics that are fed back to adaptation. It is therefore crucial to establish metrics that are meaningful for drivers as they will ultimately fuel the development of human-like self-adapting models that can be used as powerful virtual prototyping tools; i.e. that do not need simulator or real world data any more.

The time series analysis showed an initial steering overshoot in the no-steer and no-motions conditions suggesting that the driver either had an internal model that included torques/accelerations and/or was expecting a torque/acceleration build-up from the car that did not occur thus resulting in an overshoot.

The driver can of course adapt to a no-steering/no-acceleration feedback condition by simply removing those cues from his perceptual motor control interaction. The question then becomes whether the same performance can be achieved (not expected for high bandwidth manoeuvres simply because deviations from expected vehicle response cannot be detected as quickly and thus a greater error builds up) and whether it is performed with the same controller structure (drivers may adopt a new control strategy to manage the impoverished cue rendering). In either case, the metrics associated with the utility-triplet will differ significantly between reality and simulator.

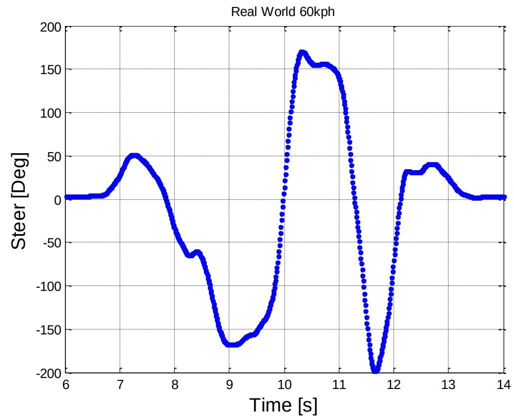

Figure 8. Real world steering angle profile of a chicane manoeuvre at 60kph.

The real-world steering profile as shown in Figure 8 shows a fundament difference with respect to the steering profiles we observed in the simulator (red lines in top panel of Figure 5). The real world steering shows an anticipatory steering action signifying either a swing wide to effectively widen the entry into the chicane or a small shift in weight balance that aids subsequent traction. Such an acausal steering action will not emerge from a pure stimulus response model; it requires a learned behaviour that results from exploration that can only be modelled by optimizing over the entire trajectory; similar to what the driver does over multiple repeated trials. This is one reason why an optimal preview control or model predictive control components has to be part of the final cybernetic model.

At the moment our model does not include such an optimal preview control component and thus it was decided not to show model results but rather focus more on understanding the observed differences in performance between the three simulator conditions.

Because our simulator driver was experienced in driving the simulator with motion and steering feedback enabled, his mental models and internal models were tuned to using these feedback cues. Without torque and acceleration feedback cues the driver has to rely on slow visual feedback cues about vehicle state and movement. Because visual feedback is slow, errors build up over a longer period and therefore to a higher magnitude. This means that the driver will exhibit larger and less frequent corrections when fast feedback is not available. This is exactly what we observed in Figure 7. The fact that corrections are less frequent is partly caused by the extra delay in using visual versus vestibular/torque feedback and partly caused by the fact that Just Noticeable Differences are higher for visual than vestibular/torque feedback.

Over time the diver is expected to adopt a different control strategy but at a high speed of 60kph he is not expected to be able to yield the same level of performance especially when normal perturbations are present such as uneven tracks in the snow or an otherwise uneven road surface. Without natural perturbations, humans are able to perform open loop control very accurately and thus it may appear as if they can yield the same performance in the simulator as in reality even with minimal feedback cues. This is why it is crucial that disturbances experienced in the real world are also represented in the simulator.

5. Conclusion

Understanding the behavioural difference between the three conditions (baseline with motion and steer torque on, no motion, and no steer feedback) requires assessment along each dimensions of the performance triplet. The aggregate performance looks at specific metrics that can be extracted from the vehicle state or driver control that quantify performance, risk or effort. Examples of these metrics are minimum proximity to the cones, standard deviation of steering angle or steering rate. These aggregate metrics can be computed from the available signals without any special decision logic. These aggregate metrics do not really show when and how control is applied. To gain insight into the specifics of control, it is necessary to explore the available time series more deeply. Examples of these metrics are peak to peak analysis in steering rate such as number of corrective steering actions and magnitude of corrective steering actions, or the lag between control actions and specific vehicle states. These metrics require development of signal specific logic to extract the meaningful time series related metrics. These time series metrics do not really show what caused the production of the specific control signal profile; to gain insight into what caused changes in control profiles, it is necessary to look at transfer functions and model coefficients.

The techniques outlined in this paper provide an objective methodology to evaluate the appropriateness of a particular simulator to appropriately characterise a specific driving task. Furthermore, the cybernetic model based approach can also provide insight into the causal mechanisms from cues to behaviour. Such a method can not only identify which modifications to an existing facility are most likely to maximise utility, but also advise on the appropriateness of simulator acquisitions or its potential to perform acceptance tests. To the best of our knowledge, no such driving simulator assessment methodology currently exists. To the question “Are we there yet?” the answer is two-fold. We believe on the one hand that the proposed approach is full of utility but on the other hand we realize that much more test track and simulator research is needed to develop necessary models.

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